$$(xy^2+3y^2)\,dy - 2x\,dx = 0, \quad y(-2)=2$$

$$(xy^2+3y^2)\frac{dy}{dx} - 2x = 0$$

$$\frac{dy}{dx} = \frac{2x}{xy^2+3y^2} = \frac{2x}{y^2(x+3)} = \underbrace{\frac{2x}{x+3}}_{g(x)} \cdot \underbrace{\frac{1}{y^2}}_{p(y)}$$

$$\int y^2\,dy = \int \frac{2x}{x+3}\,dx$$

$$x+3 \overline{)\,2x\,} = 2$$

$$\underline{-(2x+6)}$$

$$-6$$

$$2 - \frac{6}{x+3}$$

$$\frac{1}{3}y^3 = \int \left(2 - \frac{6}{x+3}\right)dx$$

$$\frac{1}{3}y^3 = 2x - 6\ln|x+3| + C$$

ARBITRARY CONSTANT

$$y^3 = 6x - 18\ln|x+3| + C$$

POSSIBLY DIFFERENT CONSTANT FROM ~~LAST~~ PREVIOUS STEP

$$y = \sqrt[3]{6x - 18\ln|x+3| + C}$$

$$2 = \sqrt[3]{-12 + C}$$

$$8 = C - 12 \rightarrow C = 20$$

$$y = \sqrt[3]{6x - 18\ln|x+3| + 20}$$

SOLVE $3vx\frac{dv}{dx}+4v^2=1$, $v(1)=-2$

IS $1-4v^2=0$ A SOL'N?

$$\frac{dv}{dx}=\frac{1-4v^2}{3vx}=\underbrace{\frac{1-4v^2}{3v}}_{p(v)}\cdot\underbrace{\frac{1}{x}}_{g(x)}$$

$u=1-4v^2$
$du=-8v\,dv$
$-\frac{3}{8}du=3v\,dv$

$$\boxed{\int\frac{3v}{1-4v^2}dv=\int\frac{1}{x}dx}$$

$$\boxed{\frac{3}{-8}\ln|1-4v^2|=\ln|x|+C}$$

$$\ln|1-4v^2|=-\frac{8}{3}\ln|x|+C$$

$$e^{\ln|1-4v^2|}=e^{-\frac{8}{3}\ln|x|+C}$$

$$=e^{-\frac{8}{3}\ln|x|}\cdot e^C$$

$\ne e^{-\frac{8}{3}\ln|x|}+e^C$

$\ne e^{-\frac{8}{3}\ln|x|}+C$

$$e^{\ln|1-4v^2|}=Ce^{-\frac{8}{3}\ln|x|}$$

$$e^{k\ln|x|}=e^{(\ln|x|)\cdot k}=(e^{\ln|x|})^k=|x|^k$$

$$\boxed{|1-4v^2|=C|x|^{-\frac{8}{3}}}$$

$$|1-4v^2|=Cx^{-\frac{8}{3}}$$

$$\boxed{1-4v^2=\cancel{\pm}Cx^{-\frac{8}{3}}}$$

$$1+Cx^{-\frac{8}{3}}=4v^2$$

$$v=\cancel{\pm}\sqrt{\frac{1+Cx^{-\frac{8}{3}}}{4}}$$

$$\boxed{v=-\sqrt{\frac{1+Cx^{-\frac{8}{3}}}{4}}}$$

$$\boxed{-2=-\sqrt{\frac{1+C}{4}}}$$

$$2=\sqrt{\frac{1+C}{4}}$$

$$4=\frac{1+C}{4}$$

$$\boxed{C=15}$$

$$\boxed{v=-\sqrt{\frac{1+15x^{-\frac{8}{3}}}{4}}}$$

IS $1-4v^2=0$ ALSO A SOL'N?

IE. IS $V=\pm\frac{1}{2}$ ALSO ~~A~~ SOL'NS? NOT TO IVP SINCE $V(1)\neq -2$

$V=\frac{1}{2}$

$\frac{dv}{dx}=0$

$V=-\frac{1}{2}$

$\frac{dv}{dx}=0$

$3vx\frac{dv}{dx}+4v^2=\cancel{3\cdot\frac{\pm 1}{2}x}\cdot 0+4\cdot\left(\frac{\pm 1}{2}\right)^2=1$

SO $V=\frac{1}{2}$ IS A SOL'N OF DE

AS IS $V=-\frac{1}{2}$ IS ALSO A SOL'N OF DE

$4v^2=1+Cx^{-\frac{8}{3}}$

$v^2=\frac{1+Cx^{-\frac{8}{3}}}{4}$

IF $C=0 \longrightarrow v^2=\frac{1}{4}$

$v=\pm\frac{1}{2}$

SO $v=\pm\frac{1}{2}$ ARE PART OF THE FAMILY OF SOL'NS

$$v^2=\frac{1+Cx^{-\frac{8}{3}}}{4}$$

2.3 LINEAR 1ST ORDER ODE.

$$a_1(x)y' + a_0(x)y = b(x) \longleftarrow a_1(x) \neq 0$$

IF $a_0(x) = 0$ $\quad a_1(x)y' = b(x)$

$$\frac{dy}{dx} = y' = \frac{b(x)}{a_1(x)}$$

$$y = \int \frac{b(x)}{a_1(x)}\,dx + C$$

IF $a_0(x) = a_1'(x)$ [IE, "COEF" OF y = DERIV OF "COEF" OF y']

EQ'N BECOMES $\underbrace{a_1(x)y' + a_1'(x)y}_{\text{PRODUCT RULE}} = b(x)$

$$\frac{d}{dx}[a_1(x)y] = b(x)$$

$$a_1(x)y = \int b(x)\,dx + C$$

DON'T USE THIS FORMULA $\quad \boxed{y = \dfrac{\int b(x)\,dx + C}{a_1(x)}}$

SOLVE $\quad x^2 \frac{dy}{dx} + 2xy = x$ $\qquad$ NOTE: $2x = (x^2)'$

$$x^2 \frac{dy}{dx} + (x^2)' y = x$$

$$x^2 y' + (x^2)' y = x$$

$$\frac{d}{dx} x^2 y = x$$

$$x^2 y = \int x \, dx + C$$

$$x^{-2}(x^2 y) = \left(\tfrac{1}{2}x^2 + C\right) x^{-2}$$

$$y = \frac{1}{2} + C x^{-2} \longrightarrow$$

CHECK:

$$\frac{dy}{dx} = -2Cx^{-3}$$

$$x^2 \frac{dy}{dx} + 2xy$$

$$= x^2(-2Cx^{-3}) + 2x\left(\tfrac{1}{2} + Cx^{-2}\right)$$

$$= \cancel{-2Cx^{-1}} + x + \cancel{2Cx^{-1}}$$

$$= x$$

SOLVE $x\left(x\,\frac{dy}{dx}+2y\right)=(1)\,x$

$$\underbrace{x^2\frac{dy}{dx}+2xy}=x$$

$\vdots$

NOTE: $2 \neq (x)'$

NOTE: $2x = (x^2)'$

HOW TO FIND THIS INTEGRATING FACTOR ?